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# Some critical remarks of recent results on F-contractions in b-metric spaces

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Abstract. In this paper, we analyze, generalize and correct some recent results on F-contractions within b-metric spaces. In all results, our only assumption is the strict growth of the function F:$(0,+\infty) \to (-\infty,+\infty)$.

## 1. Introduction and preliminaries

Generalizing Banach contraction principle [3], Wardowski [37] introduced the notion of F-contraction and proved a new fixed point theorem for it.

**Definition 1.1.** [37] Let $\mathbb{F} : (0,+\infty) \to (-\infty,+\infty)$ be a mapping satisfying the following:

(W1) $\mathbb{F}$ is strictly increasing, i.e., for all $a, b \in (0,+\infty)$ if $a < b$ then $\mathbb{F}(a) < \mathbb{F}(b)$;

(W2) For each sequence $\{a_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to+\infty} a_n = 0$ if and only if $\lim_{n\to+\infty} \mathbb{F}(a_n) = -\infty$;

(W3) There exists $\lambda \in (0,1)$ such that $\lim_{t\to 0^+} t^{\lambda}\mathbb{F}(t) = 0$.

The set of all functions satisfying the above definition Wardowski denotes with $\mathcal{F}$. The following functions $\mathbb{F}_i : (0,+\infty) \to (-\infty,+\infty)$ are in $\mathcal{F}$ : $\mathbb{F}_1(t) = \ln t$; $\mathbb{F}_2(t) = t + \ln t$; $\mathbb{F}_3(t) = -t^{-\frac{1}{2}}$; $\mathbb{F}_4(t) = \ln(t + t^2)$.

**Definition 1.2.** [37] A mapping $T : X \to X$ is said to be an F-contraction on metric space $(X, d)$ if there exist $\mathbb{F} \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,

$$d(Tx, Ty) > 0 \text{ implies } \tau + \mathbb{F}(d(Tx, Ty)) \le \mathbb{F}(d(x, y)).$$

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**Theorem 1.1.** [37] *Let $(X, d)$ be a complete metric space and let $T : X \to X$ be an $F$-contraction. Then $T$ has a unique fixed point $x^* \in X$. On the other hand, the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to $x^*$ for every $x \in X$.*

In [7], authors introduce the following condition.

(W'$_{s\tau}$): Let $\{a_n\} \subset (0, +\infty)$ be a sequence such that $\tau + \mathbb{F}(s \cdot a_n) \leq \mathbb{F}(a_{n-1})$ for all $n \in \mathbb{N}$ and for some $\tau > 0$, $s \geq 1$, then $\tau + \mathbb{F}(s^n \cdot a_n) \leq \mathbb{F}(s^{n-1} \cdot a_{n-1})$, for all $n \in \mathbb{N}$. They denote by $\mathcal{F}_s$ the family of all functions $\mathbb{F} : (0, +\infty) \to (-\infty, +\infty)$ which satisfy (W1), (W2), (W3) and (W'$_{s\tau}$). For the functions $\mathcal{F}_s$ authors in ([7], Definition 3.3., Theorem 3.4.) introduced and proved the following:

**Definition 1.3.** Let $(X, d, s \geq 1)$ be a b-metric space. A multivalued mapping $T : X \to CB(X)$ is called an $F-$contraction of Nadler type if there exist $\mathbb{F} \in \mathcal{F}_s$ and $\tau > 0$ such that

$$2\tau + \mathbb{F}(s \cdot H(Tx, Ty)) \leq \mathbb{F}(d(x, y))$$

for all $x, y \in X$ with $Tx \neq Ty$.

$CB(X)$ is the collection of all nonempty closed bounded subsets of $X$, $H(X, Y)$ is the Pompeiu–Hausdorff metric induced by $d$ for two sets $X, Y$.

**Theorem 1.2.** *Let $(X, d, s \geq 1)$ be a complete b-metric space and let $T : X \to CB(X)$. Assume that there exists a continuous function from the right $\mathbb{F} \in \mathcal{F}_s$ and $\tau > 0$ such that*

$$2\tau + \mathbb{F}(s \cdot H(Tx, Ty)) \leq \mathbb{F}(d(x, y)) \tag{1}$$

*for all $x, y \in X$ with $Tx \neq Ty$. Then $T$ has a fixed point.*

Further, in ([18], Definition 2.7.) authors introduced the following condition.

(W$_{s\tau}$): If $\inf \mathbb{F} = -\infty$ and $x, y, z \in (0, +\infty)$ are such that $\tau + \mathbb{F}(s \cdot x) \leq \mathbb{F}(y)$ and $\tau + \mathbb{F}(s \cdot y) \leq \mathbb{F}(z)$ then $\tau + \mathbb{F}(s^2 \cdot x) \leq \mathbb{F}(s \cdot y)$. Authors in [18] denote by $\mathcal{F}_{s\tau}$ the family of all functions $\mathbb{F} : (0, +\infty) \to (-\infty, +\infty)$ which satisfy (W1), (W2), (W3) and (W$_{s\tau}$).

For the functions from $\mathcal{F}_{s\tau}$ authors in ([18], Definition 3.1., Theorem 3.2.) introduced and proved the following:

**Definition 1.4.** Let $(X, d, s \geq 1)$ be a $b$-metric space and $T : X \to X$ be an operator. If there exist $\tau > 0$ and $\mathbb{F} \in \mathcal{F}_{s\tau}$ such that for all $x, y \in X$ the inequality $d(Tx, Ty) > 0$ implies

$$\tau + \mathbb{F}(s \cdot d(Tx, Ty)) \leq \mathbb{F}(d(x, y)), \tag{2}$$

then $T$ is called an $\mathbb{F}-$contraction.

**Theorem 1.3.** *Let $(X, d, s \geq 1)$ be a complete b-metric space and $T : X \to X$ be an $\mathbb{F}-$contraction, then $T$ has a unique fixed point $x^*$. Furthermore, for any $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ is convergent and $\lim_{n\to+\infty} x_n = x^*$.*

Also, for the functions from $\mathcal{F}_{s\tau}$ authors in ([18], Definition 4.1., Theorem 4.2.) introduced and proved the next:

**Definition 1.5.** Let $(X, d, s \geq 1)$ be a b-metric space and $T : X \to X$ be an operator. If there exists $\tau > 0$ and $\mathbb{F} \in \mathcal{F}_{s\tau}$ such that for all $x, y \in X$ the inequality $d(Tx, Ty) > 0$ implies

$$\tau + \mathbb{F}(s \cdot d(Tx, Ty)) \leq \mathbb{F}\left(\max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2s}\right\}\right), \tag{3}$$

then $T$ is called an $\mathbb{F}-$weak contraction.

**Theorem 1.4.** *Let $(X, d, s \geq 1)$ be a complete b-metric space and $T : X \to X$ be an $\mathbb{F}-$weak contraction, then $T$ has a unique fixed point and for any $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ is convergent in $X$. Furthermore, if either $T$ or $F$ is continuous then $T$ has a unique fixed point $x^*$ and for all $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ converges to $x^*$.*

Finally, authors in ([18], Definition 5.1., Theorem 5.2.) considered F–weak contractions of Hardy-Rogers type.

**Definition 1.6.** Let $(X, d, s \geq 1)$ be a $b$-metric space, $a, b, c, e, f \geq 0$ be real numbers and $T : X \to X$ be an operator. If there exist $\tau > 0$ and $\mathbb{F} \in \mathcal{F}_{s\tau}$ such that for all $x, y \in X$ the inequality $d(Tx, Ty) > 0$ implies

$$\tau + \mathbb{F}(s \cdot d(Tx, Ty)) \leq \mathbb{F}(ad(x, y) + bd(x, Tx) + cd(y, Ty) + ed(x, Ty) + fd(y, Tx)), \tag{4}$$

then $T$ is called an $\mathbb{F}-$weak contraction of Hardy-Rogers type.

**Theorem 1.5.** *Let $(X, d, s \geq 1)$ be a complete b-metric space and $T : X \to X$ be an $F-$weak contraction of Hardy-Rogers type. If either $a + b + c + (s + 1)e < 1$ or $a + b + c + (s + 1)f < 1$ holds then for every $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ converges to a fixed point of $T$. Moreover, if $a + e + f < s$ holds as well then $T$ has exactly one fixed point.*

First, we shall use the following two results to prove that certain Picard sequences are Cauchy in b-metric space $(X, d, s \geq 1)$. The proof is completely identical with the corresponding in [13] (see also [1]).

**Lemma 1.6.** *Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in b-metric space $(X, d, s \geq 1)$ such that*

$$d(x_n, x_{n+1}) \leq \lambda \cdot d(x_{n-1}, x_n) \tag{5}$$

*for some $\lambda \in [0, \frac{1}{s})$ and for each $n \in \mathbb{N}$. Then $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence.*

**Remark 1.7.** *It is worth noting that the previous Lemma 1.6 holds in the setting of b-metric spaces for each $\lambda \in [0, 1)$. For more details see [1].*

**Lemma 1.8.** *Let $\{x_n\}_{n\in\mathbb{N}}$ be a Picard sequence in b-metric space $(X, d, s \geq 1)$ induced by a mapping $T : X \to X$ and let $x_0 \in X$ be an initial point. If $d(x_n, x_{n+1}) < d(x_{n-1}, x_n)$ for all $n \in \mathbb{N}$ then $x_n \neq x_m$ whenever $n \neq m$.*

For more details on F-contractions the reader can find in the following recently published papers: [4], [6],[8]-[10], [14], [15], [19]-[26], [28]-[36], [38]. Also, a lot of useful things about fixed point results can be found in [1], [2], [5], [16], [17], [27], [30].

## 2. Main results

In this part of the paper we shall use only the condition (W1) for the proof of all Theorems from Section 1, Introduction and preliminaries.

Our first result refers to Theorem 1.2.

**Theorem 2.1.** *Let $(X, d, s > 1)$ be a complete b-metric space and let $T : X \to CB(X)$. Assume that there exists a strictly increasing $\mathbb{F} : (0, +\infty) \to (-\infty, +\infty)$ and $\tau > 0$ such that*

$$2\tau + \mathbb{F}(s \cdot H(Tx, Ty)) \leq \mathbb{F}(d(x, y)) \tag{6}$$

*for all $x, y \in X$ with $Tx \neq Ty$. Then $T$ has a fixed point.*

Proof. Since $\mathbb{F} : (0, +\infty) \to (-\infty, +\infty)$ is strictly increasing (satisfies (W1)) then (6) yields

$$H(Tx, Ty) < \frac{1}{s} \cdot d(x, y), \tag{7}$$

for all $x, y \in X$ with $Tx \neq Ty$. The proof further follows on the basic of Suzuki ([31], Theorems 12 and 13, Corollaries 14 and 15). See also [5] and ([17], Theorem 12.5).□

**Remark 2.2.** *Our approach significantly corrects Theorem 3.4. from [7]. Theorem 3.5. from [7] has been proved by Czerwik [5]. So no F-contraction has to be applied to its proof.*

**Remark 2.3.** *We do not know whether Theorem 2.1. is true if $s = 1$.*

Our second result contains a new approach and method of proving Theorem 1.3.

**Theorem 2.4.** *Let $(X, d, s > 1)$ be a complete b-metric space and let $T : X \to X$ be an operator. If there exist $\tau > 0$ and strictly increasing mapping $\mathbb{F} : (0, +\infty) \to (-\infty, +\infty)$ such that for all $x, y \in X$ the inequality $d(Tx, Ty) > 0$ implies*

$$\tau + F(s \cdot d(Tx, Ty)) \leq F(d(x, y)), \tag{8}$$

*then $T$ has a unique fixed point $x^*$. Furthermore, for any $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ is convergent and $\lim_{n\to+\infty} x_n = x^*$.*

PROOF. Since the function $\mathbb{F}$ is strictly increasing (8) yields

$$d(Tx,Ty) < \frac{1}{s}\cdot d(x,y), \tag{9}$$

for all $x,y \in X$ with $x \neq y$. Now condition (9) directly implies that the mapping $T$ is continuous and that its possible fixed point is unique. We did not use the function $\mathbb{F}$ as in [18] to prove uniqueness. The proof further goes on as in [13] and [17]. □

**Remark 2.5.** *As in works* [4], [9], [20],[21], [28], [33]-[36] *and here we have only used property (F1) as opposed to the approach in* [18]*. Our approach, i.e., the mode of proof is almost elementary. It follows that the introduction of $F'_{s\tau}$ and its use in* [18] *is superfluous. We see that (F4) implies the Cauchyness of the sequence $\alpha_n$. For the case $s = 1$ the result is also true (see* [36]*).*

Our next result is a generalization and correction of Theorem 1.4. from Section 1, Introduction and preliminaries of this paper.

**Theorem 2.6.** *Let $(X,d,s>1)$ be a complete b-metric space and let $T: X \to X$ be an F-weak contraction operator. If there exist $\tau > 0$ and strictly increasing mapping $\mathbb{F}: (0,+\infty) \to (-\infty,+\infty)$ such that for all $x,y \in X$ the inequality $d(Tx,Ty) > 0$ implies*

$$\begin{aligned}&\tau + \mathbb{F}(s\cdot d(Tx,Ty)) \leq \\ &\mathbb{F}\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2s}\right\}\right),\end{aligned} \tag{10}$$

*then $T$ has at most one fixed point and for any $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ is convergent in $X$. Furthermore, if either $T$ or $\mathbb{F}$ is continuous, then $T$ has a unique fixed point $x^*$ and for all $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ converges to $x^*$.*

PROOF. Since the function $\mathbb{F}$ is strictly increasing we get that (10) is equivalent to

$$s\cdot d(Tx,Ty) < \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2s}\right\}, \tag{11}$$

for all $x,y \in X$ with $x \neq y$. We prove first that $T$ has at most one fixed point. If $\overline{x},\overline{y}$ are two different fixed points of $T$, thus (11) yields

$$s\cdot d(\overline{x},\overline{y}) < \max\left\{d(\overline{x},\overline{y}), 0, 0, \frac{d(\overline{x},\overline{y})}{s}\right\} = d(\overline{x},\overline{y}), \tag{12}$$

which is a contradiction with $s > 1$.

By using (11) we shall prove that the sequence $x_{n+1} = Tx_n, n \in \mathbb{N}, x_0 \in X$ is a Cauchy sequence. Suppose that $d(x_n,x_{n+1}) > 0$ for each $n \in \mathbb{N}$. Otherwise, the proof is finished. Putting $x = x_{n-1}, y = x_n$ in (11) we get

$$s\cdot d(x_n,x_{n+1}) < \max\left\{d(x_{n-1},x_n), d(x_n,x_{n+1}), \frac{d(x_{n-1},x_{n+1})}{2s}\right\}$$

$$\leq \max\left\{d\left(x_{n-1},x_n\right),d\left(x_n,x_{n+1}\right)\right\}. \tag{13}$$

If $\max\left\{d\left(x_{n-1},x_n\right),d\left(x_n,x_{n+1}\right)\right\} = d\left(x_n,x_{n+1}\right)$ then we obtain the contradiction with $s > 1$. Hence, $\max\left\{d\left(x_{n-1},x_n\right),d\left(x_n,x_{n+1}\right)\right\} = d\left(x_{n-1},x_n\right)$. This further means that

$$d\left(x_n,x_{n+1}\right) < \frac{1}{s}d\left(x_{n-1},x_n\right) < d\left(x_{n-1},x_n\right). \tag{14}$$

According to the Lemmas 1.6 and 1.8 the condition (14) implies that $\{x_n\}$ is a Cauchy sequence as well as $x_n \neq x_m$ whenever $n \neq m$. Since $(X,d,s>1)$ is a complete b-metric space, then there exists a unique point $x^* \in X$ such that $x_n \to x^*$ as $n \to +\infty$. If the mapping $T$ is continuous we get

$$x^* = \lim_{n\to+\infty} x_n = \lim_{n\to+\infty} Tx_{n-1} = T\left(\lim_{n\to+\infty} x_{n-1}\right) = Tx^*,$$

that is., $Tx^* = x^*$.

If the function $\mathbb{F}$ is continuous the proof is identical with the corresponding in [18]. □

Finally, in the next result we generalize and correct Theorem 1.5. from Section 1, Introduction and preliminaries.

**Theorem 2.7.** *Let $(X,d,s>1)$ be a b-metric space, $a,b,c,e,f \geq 0$ be real numbers and $T : X \to X$ be an F-weak contraction of Hardy-Rogers type where $\mathbb{F} : (0,+\infty) \to (-\infty,+\infty)$ is a strictly increasing function. If $a+b+c+\frac{s+1}{2}\cdot(e+f) < 1$ holds, then for every $x_0 \in X$ the sequence $x_{n+1} = Tx_n$ converges to a fixed point of $T$. Moreover, if $a+e+f < s$ holds as well, then $T$ has exactly one fixed point.*

Proof. First of all, (4) is equivalent to

$$s \cdot d\left(Tx,Ty\right) < ad\left(x,y\right) + bd\left(x,Tx\right) + cd\left(y,Ty\right) + ed\left(x,Ty\right) + fd\left(y,Tx\right). \tag{15}$$

Let $x_0 \in X$ be an arbitrary point. If $x_k = x_{k+1}$ for some $k \in \mathbb{N}$, then $x_k$ is a (unique) fixed point of $T$ and in this case the proof is finished. Therefore, let $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Then, as in [18] by the same way we get

$$d\left(x_n,x_{n+1}\right) < \frac{1}{s}d\left(x_{n-1},x_n\right) < d\left(x_{n-1},x_n\right). \tag{16}$$

Now, according to the Lemmas 1.6 and 1.8 we get that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in a complete b-metric space $(X,d,s>1)$ as well as that $x_n \neq x_m$ whenever $n \neq m$. This means that there exists a unique $x^* \in X$ such that $x_n \to x^*$ as $n \to +\infty$. We shall show that $x^*$ is a fixed point of $T$, that is, $Tx^* = x^*$. Suppose that $Tx^* \neq x^*$. Because $x_n \neq x_m$ whenever $n \neq m$, it follows that there exists $n_0 \in \mathbb{N}$ such that $Tx^*, x^* \notin \{x_n : n \geq n_0\}$. As in ([18], line 3- on page 331) we have the following $(x = x_n, y = x^*)$

$$d(x^*,Tx^*) \le s\left[d(x^*,x_{n+1}) + d(Tx_n,Tx^*)\right]$$
$$< s\cdot d(x^*,x_{n+1}) + a\cdot d(x_n,x^*) + b\cdot d(x_n,x_{n+1}) + c\cdot d(x^*,Tx^*)$$
$$+ e\cdot d(x_n,Tx^*) + f\cdot d(x^*,x_{n+1}). \tag{17}$$

Since $d(x_n,Tx^*) \le s\left[d(x_n,x^*) + d(x^*,Tx^*)\right]$ (17) becomes

$$d(x^*,Tx^*) < s\cdot d(x^*,x_{n+1}) + a\cdot d(x_n,x^*) + b\cdot d(x_n,x_{n+1}) + c\cdot d(x^*,Tx^*)$$
$$+s\cdot e\cdot d(x_n,x^*) + s\cdot e\cdot d(x^*,Tx^*) + f\cdot d(x^*,x_{n+1}), \tag{18}$$

whenever $n \ge n_0$.

For $x = x^*, y = x_n$ we get

$$d(x^*,Tx^*) < s\cdot d(x^*,x_{n+1}) + a\cdot d(x^*,x_n) + b\cdot d(x^*,Tx^*)$$
$$+c\cdot d(x_n,x_{n+1}) + e\cdot d(x^*,x_{n+1}) + s\cdot f\cdot d(x_n,x^*) + s\cdot f\cdot d(x^*,Tx^*). \tag{19}$$

Taking the limit in (18) and (19) as $n \to +\infty$ we obtain

$$d(x^*,Tx^*) \le c\cdot d(x^*,Tx^*) + s\cdot e\cdot d(x^*,Tx^*) \le (a+b+c+(s+1)\cdot e)\cdot$$
$$\cdot\, d(x^*,Tx^*) \tag{20}$$

and

$$d(x^*,Tx^*) \le b\cdot d(x^*,Tx^*) + s\cdot f\cdot d(x^*,Tx^*) \le (a+b+c+(s+1)\cdot f)\cdot$$
$$\cdot\, d(x^*,Tx^*) \tag{21}$$

Adding (20) and (21) we get

$$d(x^*,Tx^*) \le \left(a+b+c+\frac{s+1}{2}(e+f)\right)\cdot d(x^*,Tx^*) < d(x^*,Tx^*), \tag{22}$$

which is a contradiction with $Tx^* \ne x^*$. Hence, we showed that $x^*$ is a fixed point of $T$. $\square$

**Remark 2.8.** *We have generalized Theorem 5 from [18] in several directions. For example, $\mathbb{F} : (0,+\infty) \to (-\infty,+\infty)$ satisfies only (W1) while $a+b+c+\frac{s+1}{2}\cdot(e+f) < 1$ instead of either $a+b+c+(s+1)\cdot e < 1$ or $a+b+c+(s+1)\cdot f < 1$. Instead of the approach used in [18], that Picard sequence is Cauchy, we have used Lemmas 1.6 and 1.8. We mention that Theorem 1.5. is true if $s = 1$, for details see [11]. In all the results of this work, the function $\mathbb{F}$ that maps: $(0,+\infty) \to (-\infty,+\infty)$ satisfies only property (W1). Conversely, in papers [7] and [18] it is required to satisfy all four properties (W1), (W2), (W3) and (W4). These properties were used by the authors in the mentioned works in order to prove for the defined Picard sequence that it is Cauchy. In that sense, our work generalizes and improves the results in the mentioned two works. Interestingly, many published results in some other papers can be corrected in this way.*

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