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Fixed point theorems for occasionally weakly compatible maps in neutrosophic metric spaces

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Abstract. In 2008, Al-Thagafi and Shahzad introduced the notion of Occasionally Weakly Compatible mappings (shortly OWC maps) which is more general than all the commutativity concepts. The purpose of the paper is to obtain common fixed point theorems in Neutrosophic metric spaces by using OWC maps.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [23] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michlek [10] and George and Veeramani [6] modified the notion of fuzzy metric spaces with the help of continuous t-norms. As a generalization of fuzzy sets, Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets. Park [13] using the idea of intuitionistic fuzzy sets defined the notion of Intuitionistic Fuzzy Metric Spaces (IFMS) with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric spaces. The concept of compatible maps and weakly compatible maps in fuzzy metric space was introduced by Jungck [7]. Later it was generalized by Al. Thagafi et al. [4] by introducing the concept of occasionally weakly compatible mappings. Saadati et al. [18] introduced the modified intuitionistic fuzzy metric space and proved some fixed point theorems for compatible and weakly compatible maps. Several researchers extend and compliments many results existing in the literature including those of Aliouche [2] and Bouhadjera [5]. More

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recently, Abbas and Rhoades [1] extended the definition of occasionally weakly compatible maps to the setting of single valued maps and they proved some common fixed point theorems satisfying generalized contractive conditions of integral type. Samarandache [17] introduced Neutrosophic set, which is a generalized of fuzzy and intuitionistic fuzzy set by incorporating a degree of indeterminacy. In 2019, Kirisci et al. [11] defined Neutrosophic Metric Space (NMS) as a generalization of IFMS and brings about fixed point theorems in complete NMS. Later, Sowndrarajan et al. [19] proved some fixed point results for contraction theorems in NMS. In this paper, we have proved common fixed point theorems in neutrosophic metric spaces by using OWC maps.

2. Preliminaries

Definition 2.1. [7] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm [CTN] if it satisfies the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $\varepsilon_1 * 1 = \varepsilon_1$ for all $\varepsilon_1 \in [0,1]$,
(iv) $\varepsilon_1 * \varepsilon_2 \leq \varepsilon_3 * \varepsilon_4$ whenever $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$, for each $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0,1]$.

Definition 2.2. [7] A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm [CTC] if it satisfies the following conditions :
(i) $\Diamond$ is commutative and associative,
(ii) $\Diamond$ is continuous,
(iii) $\varepsilon_1 \Diamond 0 = \varepsilon_1$ for all $\varepsilon_1 \in [0,1]$,
(iv) $\varepsilon_1 \Diamond \varepsilon_2 \leq \varepsilon_3 \Diamond \varepsilon_4$ whenever $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$, for each $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\varepsilon_4 \in [0,1]$.

Definition 2.3. [20] A 6-tuple $(\sum, \Xi, \Theta, \Upsilon, *, \Diamond)$ is said to be an NMS, if $\sum$ is an arbitrary non empty set, $*$ is a neutrosophic CTN, $\Diamond$ is a neutrosophic CTC and Ξ, Θ and Υ are neutrosoophic $\sum^3 \times \mathbb{R}^+$ satisfying the following conditions: For all $\zeta, \eta, \delta, \omega \in \sum, \lambda \in \mathbb{R}^+$.
(i) $0 \leq \Xi(\zeta, \eta, \delta, \lambda) \leq 1; 0 \leq \Theta(\zeta, \eta, \delta, \lambda) \leq 1; 0 \leq \Upsilon(\zeta, \eta, \delta, \lambda) \leq 1$;
(ii) $\Xi(\zeta, \eta, \delta, \lambda) + \Theta(\zeta, \eta, \delta, \lambda) + \Upsilon(\zeta, \eta, \delta, \lambda) \leq 3$;
(iii) $\Xi(\zeta, \eta, \delta, \lambda) = 1$ if and only if $\zeta = \eta = \delta$;
(iv) $\Xi(\zeta, \eta, \delta, \lambda) = \Xi(\wp(\zeta, \eta, \delta, \lambda))$, when $\wp$ is the permutation function;
(v) $\Xi(\zeta, \eta, \omega, \lambda) * \Xi(\omega, \delta, \delta, \mu) \leq \Xi(\zeta, \eta, \delta, \lambda + \mu)$, for all $\lambda, \mu > 0$;
(vi) $\Xi(\zeta, \eta, \delta, .) : [0, \infty) \to [0,1]$ is neutrosophic continuous;
(vii) $\lim_{\lambda \to \infty} \Xi(\zeta, \eta, \delta, \lambda) = 1$ for all $\lambda > 0$;
(viii) $\Theta(\zeta, \eta, \delta, \lambda) = 0$ if and only if $\zeta = \eta = \delta$;
(ix) $\Theta(\zeta, \eta, \delta, \lambda) = \Theta(\wp(\zeta, \eta, \delta, \lambda))$, when $\wp$ is the permutation function;
(x) $\Theta(\zeta, \eta, \omega, \lambda) \Diamond \Theta(\omega, \delta, \delta, \mu) \geq \Theta(\zeta, \eta, \delta, \lambda + \mu)$, for all $\lambda, \mu > 0$;
(xi) $\Theta(\zeta, \eta, \delta, .) : [0, \infty) \to [0,1]$ is neutrosophic continuous ;

(xii) $\lim_{\lambda\to\infty} \Theta(\zeta,\eta,\delta,\lambda) = 0$ for all $\lambda > 0$;
(xiii) $\Upsilon(\zeta,\eta,\delta,\lambda) = 0$ if and only if $\zeta = \eta = \delta$;
(xiv) $\Upsilon(\zeta,\eta,\delta,\lambda) = \Upsilon(\wp(\zeta,\eta,\delta,\lambda))$, when $\wp$ is the permutation function;
(xv) $\Upsilon(\zeta,\eta,\omega,\lambda)\Diamond\Upsilon(\omega,\delta,\delta,\mu) \geq \Upsilon(\zeta,\eta,\delta,\lambda+\mu)$, for all $\lambda,\mu > 0$;
(xvi) $\Upsilon(\zeta,\eta,\delta,.) : [0,\infty) \to [0,1]$ is neutrosophic continuous;
(xvii) $\lim_{\lambda\to\infty} \Upsilon(\zeta,\eta,\delta,\lambda) = 0$ for all $\lambda > 0$;
(xviii) If $\lambda \leq 0$ then $\Xi(\zeta,\eta,\delta,\lambda) = 0; \Theta(\zeta,\eta,\delta,\lambda) = 1; \Upsilon(\zeta,\eta,\delta,\lambda) = 1$.
Then, $(\sum,\Xi,\Theta,\Upsilon)$ is called an NMS on $\sum$. The function Ξ,Θ and Υ denote degree of closedness, neturalness and non-closedness between ζ,η and δ with respect to λ respectively.

Example 2.4. [20] Let $(\sum, D)$ be a metric space. Define $\omega * \tau = \min\{\omega,\tau\}$ and $\omega\Diamond\tau = \max\{\omega,\tau\}$ and $\Xi,\Theta,\Upsilon : \sum^3 \times\mathbb{R}^+ \to [0,1]$ defined by, we define

$$\Xi(\zeta,\eta,\delta,\lambda) = \frac{\lambda}{\lambda + D(\zeta,\eta,\delta)}; \Theta(\zeta,\eta,\delta,\lambda) = \frac{D(\zeta,\eta,\delta)}{\lambda + D(\zeta,\eta,\delta)}; \Upsilon(\zeta,\eta,\delta,\lambda) = \frac{D(\zeta,\eta,\delta)}{\lambda}$$

for all $\zeta,\eta,\delta \in \sum$ and $\lambda > 0$. Then $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ is called NMS induced by a metric D the standard neutrosophic metric.

Lemma 2.1. *Let $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ be an NMS. Then $\Xi(\zeta,\eta,\delta,\lambda)$ is non-decreasing and $\Theta(\zeta,\eta,\delta,\lambda), \Upsilon(\zeta,\eta,\delta,\lambda)$ are non-increasing with respect to λ, for all $\zeta,\eta,\delta \in \sum$.*

Definition 2.5. Let $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ be an NMS and $\{\zeta_n\}$ be a sequence in $\sum$. $\{\zeta_n\}$ is said to be converges to a point $\zeta \in \sum$ if
(a) $\lim_{n\to\infty} \Xi(\zeta,\zeta,\zeta_n,\lambda) = 1$, $\lim_{n\to\infty} \Theta(\zeta,\zeta,\zeta_n,\lambda) = 0$, $\lim_{n\to\infty} \Upsilon(\zeta,\zeta,\zeta_n,\lambda) = 0$, for all $\lambda > 0$.
(b) $\{\zeta_n\}$ is called Cauchy sequence if $\lim_{n\to\infty} \Xi(\zeta_{n+p},\zeta_{n+p},\zeta_n,\lambda) = 1$, $\lim_{n\to\infty} \Theta(\zeta_{n+p},\zeta_{n+p},\zeta_n,\lambda) = 0$ and $\lim_{n\to\infty} \Upsilon(\zeta_{n+p},\zeta_{n+p},\zeta_n,\lambda) = 0$, for all $\lambda > 0$ and $p > 0$.
(c) A NMS in which every Cauchy sequence is convergent is said to be complete.

Definition 2.6. Two self mappings Γ and ω of NMS $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ are called compatible if $\lim_{n\to\infty} \Xi(\Gamma\omega\zeta_n,\omega\Gamma\zeta_n,\omega\Gamma\zeta_n,\lambda) = 1$, $\lim_{n\to\infty} \Theta(\Gamma\omega\zeta_n,\omega\Gamma\zeta_n,\omega\Gamma\zeta_n,\lambda) = 0$ and $\lim_{n\to\infty} \Upsilon(\Gamma\omega\zeta_n,\omega\Gamma\zeta_n,\omega\Gamma\zeta_n,\lambda) = 0$, whenever $\{\zeta_n\}$ is a sequence in $\sum$ such that $\lim_{n\to\infty} \Gamma\zeta_n = \lim_{n\to\infty} \Gamma\zeta_n = \zeta$, for some $\zeta \in \sum$.

Definition 2.7. Let Γ and ω be maps from an NMS $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ into itself. The maps Γ and ω are said to be OWC if and only if there is a point $\zeta \in \sum$ which is a coincidence point of Γ and ω at which $\mathcal{A}$ and $\mathcal{S}$ commute i.e., there is a point $\zeta \in \sum$ such that $\Gamma\zeta = \omega\zeta$ and $\Gamma\omega\zeta = \omega\Gamma\zeta$.

Lemma 2.2. *Let $\sum$ be a set Γ and ω OWC self maps of $\sum$. If Γ and ω have a unique point of coincidence, $\omega = \Gamma\zeta = \omega\zeta$, then ω is the unique common fixed point of Γ and ω.*

Example 2.8. Let $\Sigma = [0, \infty)$ with the metric d is defined by $d(\zeta, \eta) = |\zeta - \eta|$. Where $\star$ and $\Diamond$ defined by $a \star b = min\{a, b\}, \quad a \Diamond b = max\{a, b\}$. we define (Ξ, Θ, Υ) by

$$\Xi(\zeta, \eta, \lambda) = \frac{\lambda}{\lambda + d(\zeta, \eta)}; \quad \Theta(\zeta, \eta, \lambda) \quad = \frac{d(\zeta, \eta)}{\lambda + d(\zeta, \eta)}; \quad \Upsilon(\zeta, \eta, \lambda) = \frac{d(\zeta, \eta)}{\lambda}.$$

for all $\zeta, \eta \in \Sigma$ and $\lambda > 0$. Then $(\Sigma, \Xi, \Theta, \Upsilon, \star, \diamond)$ is a neutrosophic metric space. Let Γ, ω be two self maps on $\sum$ defined by

$$\Gamma(\zeta) = \sqrt{\frac{1 - (2\zeta - 1)^2}{2}}; \quad \omega(\zeta) = 1 - \zeta$$

Here Γ, ω have two coincidence fixed points $\zeta = 1$, and $\zeta = \frac{1}{2}$, since $\Gamma(1) = \omega(1) = 0$ for $\zeta = 1$. Also, for $\zeta = \frac{1}{2}$ we get, $\Gamma(\frac{1}{2}) = \omega(\frac{1}{2}) = \frac{1}{2}$ where $\zeta = \frac{1}{2}$ is a common fixed point.
So Γ, ω are OWC maps, since they commute at one of their coincidence point $\zeta = \frac{1}{2}$.

3. Main Results

Theorem 3.1. *Let $(\sum, \Xi, \Theta, \Upsilon, *, \Diamond)$ be the complete NMS and let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ be self mapping of $\sum$. Let the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$ be OWC and $\rho > 1$, then*

$$\Xi(\mathcal{A}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \rho\lambda) \leq \min \left\{ \begin{array}{c} \left(\begin{array}{c} \Xi(\mathcal{S}\zeta, \mathcal{T}\eta, \mathcal{T}\eta, \lambda), \Xi(\mathcal{S}\zeta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda), \Xi(\mathcal{T}\eta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \\ \Xi(\mathcal{S}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \Xi(\mathcal{T}\eta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda) \end{array} \right) \\ \frac{\alpha\Xi(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Xi(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}, \frac{1+\Xi(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2} \end{array} \right\}$$

$$\Theta(\mathcal{A}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \left(\begin{array}{c} \Theta(\mathcal{S}\zeta, \mathcal{T}\eta, \mathcal{T}\eta, \lambda), \Theta(\mathcal{S}\zeta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda), \Theta(\mathcal{T}\eta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \\ \Theta(\mathcal{S}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \Theta(\mathcal{T}\eta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda) \end{array} \right) \\ \frac{\alpha\Theta(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Theta(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}, \frac{1+\Theta(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2} \end{array} \right\}$$

$$\Upsilon(\mathcal{A}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \left(\begin{array}{c} \Upsilon(\mathcal{S}\zeta, \mathcal{T}\eta, \mathcal{T}\eta, \lambda), \Upsilon(\mathcal{S}\zeta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda), \Upsilon(\mathcal{T}\eta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \\ \Upsilon(\mathcal{S}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \Upsilon(\mathcal{T}\eta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda) \end{array} \right) \\ \frac{\alpha\Upsilon(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Upsilon(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}, \frac{1+\Upsilon(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2} \end{array} \right\} \tag{3.1.1}$$

for all $\zeta, \eta \in \sum$ and $\lambda > 0$ such that $\mathcal{A}\omega = \mathcal{S}\omega = \omega$ and a unique point $\delta \in \sum$ such that $\mathcal{B}\delta = \mathcal{T}\delta = \delta$. Moreover $\delta = \omega$, so that there is a unique common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$.

Proof. Let the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$ are OWC so there are points $\zeta, \eta \in \sum$ such that $\mathcal{A}\zeta = \mathcal{S}\zeta$ and $\mathcal{B}\eta = \eta$, We claim that $\mathcal{A}\zeta = \mathcal{B}\eta$. If not then by inequality (3.1.1)

$$\Xi(\mathcal{A}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \rho\lambda) \leq \min \left\{ \begin{array}{c} \left(\begin{array}{c} \Xi(\mathcal{S}\zeta, \mathcal{T}\eta, \mathcal{T}\eta, \lambda), \Xi(\mathcal{S}\zeta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda), \Xi(\mathcal{T}\eta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \\ \Xi(\mathcal{S}\zeta, \mathcal{B}\eta, \mathcal{B}\eta, \lambda), \mathcal{M}(\mathcal{T}\eta, \mathcal{A}\zeta, \mathcal{A}\zeta, \lambda) \end{array} \right) \\ \frac{\alpha\Xi(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Xi(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}, \frac{1+\Xi(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2} \end{array} \right\}$$

$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\leq \min\{\Xi(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),1,1,\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),1\}$

$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\leq \Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)$,

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Theta(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Theta(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Theta(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}$$

$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \max\{\Theta(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),0,0,\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),0\}$

$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)$ and

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Upsilon(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Upsilon(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Upsilon(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}$$

$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \max\{\Upsilon(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),0,0,\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),0\}$

$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)$.

Then by lemma (2.9), $\mathcal{A}\zeta=\mathcal{B}\eta$. Suppose that there is another point δ such that $\mathcal{A}\delta=\mathcal{S}\delta$.
Then by inequality (3.1.1), we have $\mathcal{A}\delta=\mathcal{S}\delta=\mathcal{B}\eta=\mathcal{T}\eta$.
So, $\mathcal{A}\zeta=\mathcal{A}\delta$ and $\omega=\mathcal{A}\zeta=\mathcal{S}\zeta$ is the unique point of coincidence of $\mathcal{A}$ and $\mathcal{S}$.
By lemma (2.9), ω is the only common point of $\mathcal{A}$ and $\mathcal{S}$.
Similarly , there is a unique point $\delta\in\sum$ such that $\delta=\mathcal{B}\delta=\mathcal{T}\delta$.
Assume that $\omega\neq\delta$, then by (3.1.1), $\Xi(\omega,\delta,\delta,\rho\lambda)=\Xi(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\leq \min\left\{\begin{array}{c}\left(\begin{array}{c}\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Xi(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Xi(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Xi(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Xi(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Xi(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}$$

$$\Xi(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)\leq \min\left\{\begin{array}{c}\left(\begin{array}{c}\Xi(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda),\Xi(\mathcal{S}\omega,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),\Xi(\mathcal{T}\delta,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Xi(\mathcal{S}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Xi(\mathcal{T}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda)\end{array}\right)\\ \frac{\alpha\Xi(\mathcal{A}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)+\beta\Xi(\mathcal{B}\delta,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)+\Gamma\Xi(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Xi(\mathcal{A}\omega,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)}{2}\end{array}\right\}$$

$\Xi(\omega,\delta,\delta,\rho\lambda)\leq \min\{\Xi(\omega,\delta,\delta,\lambda),1,1,\Xi(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Xi(\mathcal{B}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),1\}$

$\Xi(\omega,\delta,\delta,\rho\lambda)\leq \Xi(\omega,\delta,\delta,\lambda)$,

$\Theta(\omega,\delta,\delta,\rho\lambda)=\Theta(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Theta(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Theta(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Theta(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}$$

$$\Theta(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)\geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Theta(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda),\Theta(\mathcal{S}\omega,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),\Theta(\mathcal{T}\delta,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Theta(\mathcal{S}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Theta(\mathcal{T}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda)\end{array}\right)\\ \frac{\alpha\Theta(\mathcal{A}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)+\beta\Theta(\mathcal{B}\delta,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)+\Gamma\Theta(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Theta(\mathcal{A}\omega,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)}{2}\end{array}\right\}$$

$$\Theta(\omega,\delta,\delta,\rho\lambda) \geq \max\{\Theta(\omega,\delta,\delta,\lambda),0,0,\Theta(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Theta(\mathcal{B}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),0\}.$$
$$\Theta(\omega,\delta,\delta,\rho\lambda) \geq \Theta(\omega,\delta,\delta,\lambda) \quad \text{and}$$
$$\Upsilon(\omega,\delta,\delta,\rho\lambda) = \Upsilon(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$$
$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda) \geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Upsilon(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Upsilon(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Upsilon(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}$$
$$\Upsilon(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda) \geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Upsilon(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda),\Upsilon(\mathcal{S}\omega,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),\Upsilon(\mathcal{T}\delta,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Upsilon(\mathcal{S}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Upsilon(\mathcal{T}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda)\end{array}\right)\\ \frac{\alpha\Upsilon(\mathcal{A}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)+\beta\Upsilon(\mathcal{B}\delta,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)+\Gamma\Upsilon(\mathcal{S}\omega,\mathcal{T}\delta,\mathcal{T}\delta,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Upsilon(\mathcal{A}\omega,\mathcal{S}\omega,\mathcal{S}\omega,\lambda)}{2}\end{array}\right\}$$
$$\Upsilon(\omega,\delta,\delta,\rho\lambda) \geq \max\{\Upsilon(\omega,\delta,\delta,\lambda),0,0,\Upsilon(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\Upsilon(\mathcal{B}\delta,\mathcal{A}\omega,\mathcal{A}\omega,\lambda),0\}.$$
$$\Upsilon(\omega,\delta,\delta,\rho\lambda) \geq \Upsilon(\omega,\delta,\delta,\lambda).$$

Then by lemma (2.9). Therefore $\omega=\delta$. δ is a common fixed point of $\mathcal{A},\mathcal{B},\mathcal{S}$ and $\mathcal{T}$.
Uniqueness:
Let μ be another common fixed point of $\mathcal{A},\mathcal{B},\mathcal{S}$ and $\mathcal{T}$. Then, put $\zeta=\delta$ and $\eta=\mu$ in (3.1.1),

$$\Xi(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \leq \min\left\{\begin{array}{c}\left(\begin{array}{c}\Xi(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda),\Xi(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda),\Xi(\mathcal{T}\mu,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Xi(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda),\Xi(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda)\end{array}\right)\\ \frac{\alpha\Xi(\mathcal{A}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)+\beta\Xi(\mathcal{B}\mu,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)+\Gamma\Xi(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Xi(\mathcal{A}\delta,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)}{2}\end{array}\right\}$$
$$\Xi(\delta,\mu,\mu,\rho\lambda) \leq \min\{\Xi(\delta,\mu,\mu,\lambda),1,1,\Xi(\delta,\mu,\mu,\lambda),\Xi(\mu,\delta,\delta,\lambda),1\}$$
$$\Xi(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \leq \Xi(\delta,\mu,\mu,\lambda),$$
$$\Theta(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Theta(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda),\Theta(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda),\Theta(\mathcal{T}\mu,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Theta(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda),\Theta(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda)\end{array}\right)\\ \frac{\alpha\Theta(\mathcal{A}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)+\beta\Theta(\mathcal{B}\mu,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)+\Gamma\Theta(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Theta(\mathcal{A}\delta,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)}{2}\end{array}\right\}$$
$$\Theta(\delta,\mu,\mu,\rho\lambda) \geq \max\{\Theta(\delta,\mu,\mu,\lambda),0,0,\Theta(\delta,\mu,\mu,\lambda),\Theta(\mu,\delta,\delta,\lambda),0\}$$
$$\Theta(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \Theta(\delta,\mu,\mu,\lambda) \quad \text{and}$$
$$\Upsilon(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\Upsilon(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda),\Upsilon(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda),\Upsilon(\mathcal{T}\mu,\mathcal{B}\delta,\mathcal{B}\delta,\lambda),\\ \Upsilon(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda),\Upsilon(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda)\end{array}\right)\\ \frac{\alpha\Upsilon(\mathcal{A}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)+\beta\Upsilon(\mathcal{B}\mu,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)+\Gamma\Upsilon(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda)}{\alpha+\beta+\Gamma},\frac{1+\Upsilon(\mathcal{A}\delta,\mathcal{S}\delta,\mathcal{S}\delta,\lambda)}{2}\end{array}\right\}$$
$$\Upsilon(\delta,\mu,\mu,\rho\lambda) \geq \max\{\Upsilon(\delta,\mu,\mu,\lambda),0,0,\Upsilon(\delta,\mu,\mu,\lambda),\Upsilon(\mu,\delta,\delta,\lambda),0\},$$
$$\Upsilon(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \Upsilon(\delta,\mu,\mu,\lambda).$$

Then by lemma (2.9) $\delta=\mu$. □

Theorem 3.2. *Let* $(\sum,\Xi,\Theta,\Upsilon,*,\Diamond)$ *be the complete NMS and let* $\mathcal{A},\mathcal{B},\mathcal{S}$ *and* $\mathcal{T}$ *be self mapping of* $\sum$. *Let the pairs* $(\mathcal{A},\mathcal{S})$ *and* $(\mathcal{B},\mathcal{T})$ *be OWC and* $\rho>1$ *and*

$\alpha+\beta=1$, then

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\le\min\left\{\begin{array}{c}\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Xi(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\min\left\{\begin{array}{c}\Xi(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\},$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\ge\max\left\{\begin{array}{c}\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Theta(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\ge\max\left\{\begin{array}{c}\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Upsilon(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\} \tag{3.2.1}$$

for all $\zeta,\eta\in\sum$ and $\lambda>0$ such that $\mathcal{A}\omega=\mathcal{S}\omega=\omega$ and a unique point $\delta\in\sum$ such that $\mathcal{B}\delta=\mathcal{T}\delta=\delta$. Moreover $\delta=\omega$, so, that there is unique fixed point of $\mathcal{A},\mathcal{B},\mathcal{S}$ and $\mathcal{T}$

Proof. Let the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ are OWC so there are points $\zeta,\eta\in\sum$ such that $\mathcal{A}\zeta=\mathcal{S}\zeta$ and $\mathcal{B}\eta=\mathcal{T}\eta$. We claim that $\mathcal{A}\zeta=\mathcal{B}\eta$. If not then by inequality (3.2.1),

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\le\min\left\{\begin{array}{c}\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Xi(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\min\left\{\begin{array}{c}\Xi(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Xi(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\le\min\left\{\begin{array}{c}\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),1,1,\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \alpha\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)+\beta\min\{1,1,\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}\end{array}\right\}$$

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\le\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\ge\max\left\{\begin{array}{c}\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Theta(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Theta(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\max\left\{\begin{array}{c}\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),0,0,\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \alpha\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)+\beta\max\{0,0,\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}\end{array}\right\}$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\quad\text{and}$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\max\left\{\begin{array}{c}\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Upsilon(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Upsilon(\mathcal{B}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{A}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\max\left\{\begin{array}{c}\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),0,0,\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{B}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \alpha\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)+\beta\max\{0,0,\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}\end{array}\right\}$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda).$$

Then by lemma (2.9), $\mathcal{A}\zeta=\mathcal{B}\eta$.
Suppose that there is another point δ such that $\mathcal{A}\delta=\mathcal{S}\delta$.
Then by inequality (3.2.1), we have $\mathcal{A}\delta=\mathcal{S}\delta=\mathcal{B}\eta=\mathcal{T}\eta$, So, $\mathcal{A}\zeta=\mathcal{A}\delta$ and $\omega=\mathcal{A}\zeta=\mathcal{S}\zeta$ is the unique point of coincidence of $\mathcal{A}$ and $\mathcal{S}$. By lemma (2.9), ω is the only common point of $\mathcal{A}$ and $\mathcal{S}$.
Similarly there is a unique point $\delta\in\sum$ such that $\delta=\mathcal{B}\delta=\mathcal{T}\delta$.
Assume that $\omega\neq\delta$, then by (3.2.1),

$$\Xi(\omega,\delta,\delta,\rho\lambda)=\Xi(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$$

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\leq\min\left\{\begin{array}{c}\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Xi(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Xi(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\min\left\{\begin{array}{c}\Xi(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\},$$

$$\Theta(\omega,\delta,\delta,\rho\lambda)=\Theta(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\max\left\{\begin{array}{c}\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Theta(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

$$\text{and}\quad\Upsilon(\omega,\delta,\delta,\rho\lambda)=\Upsilon(\mathcal{A}\omega,\mathcal{B}\delta,\mathcal{B}\delta,\rho\lambda)$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)\geq\max\left\{\begin{array}{c}\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \{\alpha\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\}+\beta\max\left\{\begin{array}{c}\Upsilon(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)\end{array}\right\}\end{array}\right\}$$

Put $\zeta = \omega$ and $\eta = \delta$ in inequality (3.2.1),

$$\Xi(\mathcal{A}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \rho\lambda) \leq \min \left\{ \begin{array}{c} \Xi(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \Xi(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \Xi(\mathcal{T}\delta, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \\ \Xi(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \Xi(\mathcal{T}\delta, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \{\alpha\Xi(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda)\} + \beta \min \left\{ \begin{array}{c} \Xi(\mathcal{B}\delta, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \\ \Xi(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \Xi(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Xi(\omega, \delta, \delta, \rho\lambda) \leq \min \left\{ \begin{array}{c} \Xi(\omega, \delta, \delta, \lambda), \Xi(\omega, \omega, \omega, \lambda), \Xi(\delta, \delta, \delta, \lambda), \\ \Xi(\omega, \delta, \delta, \lambda), \Xi(\delta, \omega, \omega, \lambda), \\ \{\alpha\Xi(\omega, \delta, \delta, \lambda)\} + \beta \min \left\{ \begin{array}{c} \Xi(\delta, \delta, \delta, \lambda), \\ \Xi(\omega, \omega, \omega, \lambda), \\ \Xi(\omega, \delta, \delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Xi(\omega, \delta, \delta, \rho\lambda) \leq \min \left\{ \begin{array}{c} \Xi(\omega, \delta, \delta, \lambda), 1, 1, \Xi(\delta, \omega, \omega, \lambda), \\ \{\alpha\Xi(\omega, \delta, \delta, \lambda)\} + \beta \min\{1, 1, \Xi(\omega, \delta, \delta, \lambda)\} \end{array} \right\}$$

$$\Xi(\omega, \delta, \delta, \rho\lambda) \leq \Xi(\omega, \delta, \delta, \rho\lambda),$$

$$\Theta(\mathcal{A}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \Theta(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \Theta(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \Theta(\mathcal{T}\delta, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \\ \Theta(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \Theta(\mathcal{T}\delta, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \{\alpha\Theta(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda)\} + \beta \max \left\{ \begin{array}{c} \Theta(\mathcal{B}\delta, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \\ \Theta(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \Theta(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Theta(\omega, \delta, \delta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \Theta(\omega, \delta, \delta, \lambda), \Theta(\omega, \omega, \omega, \lambda), \Theta(\delta, \delta, \delta, \lambda), \\ \Theta(\omega, \delta, \delta, \lambda), \Theta(\delta, \omega, \omega, \lambda), \\ \{\alpha\Theta(\omega, \delta, \delta, \lambda)\} + \beta \max \left\{ \begin{array}{c} \Theta(\delta, \delta, \delta, \lambda), \\ \Theta(\omega, \omega, \omega, \lambda), \\ \Theta(\omega, \delta, \delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Theta(\omega, \delta, \delta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \Theta(\omega, \delta, \delta, \lambda), 0, 0, \Theta(\delta, \omega, \omega, \lambda), \\ \{\alpha\Theta(\omega, \delta, \delta, \lambda)\} + \beta \max\{0, 0, \Theta(\omega, \delta, \delta, \lambda)\} \end{array} \right\}$$

$$\Theta(\omega, \delta, \delta, \rho\lambda) \geq \Theta(\omega, \delta, \delta, \rho\lambda) \quad \text{and}$$

$$\Upsilon(\mathcal{A}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \Upsilon(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \Upsilon(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \Upsilon(\mathcal{T}\delta, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \\ \Upsilon(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda), \Upsilon(\mathcal{T}\delta, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \{\alpha\Upsilon(\mathcal{S}\omega, \mathcal{B}\delta, \mathcal{B}\delta, \lambda)\} + \beta \max \left\{ \begin{array}{c} \Upsilon(\mathcal{B}\delta, \mathcal{T}\delta, \mathcal{T}\delta, \lambda), \\ \Upsilon(\mathcal{S}\omega, \mathcal{A}\omega, \mathcal{A}\omega, \lambda), \\ \Upsilon(\mathcal{S}\omega, \mathcal{T}\delta, \mathcal{T}\delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Upsilon(\omega, \delta, \delta, \rho\lambda) \geq \max \left\{ \begin{array}{c} \Upsilon(\omega, \delta, \delta, \lambda), \Upsilon(\omega, \omega, \omega, \lambda), \Upsilon(\delta, \delta, \delta, \lambda), \\ \Upsilon(\omega, \delta, \delta, \lambda), \Upsilon(\delta, \omega, \omega, \lambda), \\ \{\alpha\Upsilon(\omega, \delta, \delta, \lambda)\} + \beta \max \left\{ \begin{array}{c} \Upsilon(\delta, \delta, \delta, \lambda), \\ \Upsilon(\omega, \omega, \omega, \lambda), \\ \Upsilon(\omega, \delta, \delta, \lambda) \end{array} \right\} \end{array} \right\}$$

$$\Upsilon(\omega,\delta,\delta,\rho\lambda) \geq \max\left\{\begin{array}{c} \Upsilon(\omega,\delta,\delta,\lambda), 0, 0, \Upsilon(\delta,\omega,\omega,\lambda), \\ \{\alpha\Upsilon(\omega,\delta,\delta,\lambda)\} + \beta\max\{0,0,\Upsilon(\omega,\delta,\delta,\lambda)\} \end{array}\right\}$$
$$\Upsilon(\omega,\delta,\delta,\rho\lambda) \geq \Upsilon(\omega,\delta,\delta,\rho\lambda).$$

Then by lemma (2.9), we get $\omega = \delta$.
Uniqueness: Let μ be another common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$. Then, put $\zeta = \delta$ and $\eta = \mu$ in (3.2.1),

$$\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda) \leq \min\left\{\begin{array}{c} \Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \Xi(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \\ \Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \Xi(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \{\alpha\Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\} + \beta\min\left\{\begin{array}{c} \Xi(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \\ \Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Xi(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \leq \min\left\{\begin{array}{c} \Xi(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \Xi(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \Xi(\mathcal{T}\mu,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \\ \Xi(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \Xi(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \{\alpha\Xi(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda)\} + \beta\min\left\{\begin{array}{c} \Xi(\mathcal{B}\mu,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \\ \Xi(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \Xi(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Xi(\delta,\mu,\mu,\rho\lambda) \leq \min\left\{\begin{array}{c} \Xi(\delta,\mu,\mu,\lambda), \Xi(\delta,\delta,\delta,\lambda), \Xi(\mu,\mu,\mu,\lambda), \\ \Xi(\delta,\mu,\mu,\lambda), \Xi(\mu,\delta,\delta,\lambda), \\ \{\alpha\Xi(\delta,\mu,\mu,\lambda)\} + \beta\min\left\{\begin{array}{c} \Xi(\mu,\mu,\mu,\lambda), \\ \Xi(\delta,\delta,\delta,\lambda), \\ \Xi(\delta,\mu,\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Xi(\delta,\mu,\mu,\rho\lambda) \leq \min\left\{\begin{array}{c} \Xi(\delta,\mu,\mu,\lambda), 1, 1, \Xi(\delta,\mu,\mu,\lambda), \Xi(\mu,\delta,\delta,\lambda), \\ \{\alpha\Xi(\delta,\mu,\mu,\lambda)\} + \beta\min\{1,1,\Xi(\delta,\mu,\mu,\lambda)\} \end{array}\right\}$$
$$\Xi(\delta,\mu,\mu,\rho\lambda) \leq \Xi(\delta,\mu,\mu,\lambda),$$

$$\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda) \geq \max\left\{\begin{array}{c} \Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \\ \Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \{\alpha\Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\} + \beta\max\left\{\begin{array}{c} \Theta(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \\ \Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Theta(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Theta(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \Theta(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \Theta(\mathcal{T}\mu,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \\ \Theta(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \Theta(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \{\alpha\Theta(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda)\} + \beta\max\left\{\begin{array}{c} \Theta(\mathcal{B}\mu,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \\ \Theta(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \Theta(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Theta(\delta,\mu,\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Theta(\delta,\mu,\mu,\lambda), \Theta(\delta,\delta,\delta,\lambda), \Theta(\mu,\mu,\mu,\lambda), \\ \Theta(\delta,\mu,\mu,\lambda), \Theta(\mu,\delta,\delta,\lambda), \\ \{\alpha\Theta(\delta,\mu,\mu,\lambda)\} + \beta\max\left\{\begin{array}{c} \Theta(\mu,\mu,\mu,\lambda), \\ \Theta(\delta,\delta,\delta,\lambda), \\ \Theta(\delta,\mu,\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Theta(\delta,\mu,\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Theta(\delta,\mu,\mu,\lambda), 0, 0, \Theta(\delta,\mu,\mu,\lambda), \Theta(\mu,\delta,\delta,\lambda), \\ \{\alpha\Theta(\delta,\mu,\mu,\lambda)\} + \beta\max\{0,0,\Theta(\delta,\mu,\mu,\lambda)\} \end{array}\right\}$$

$$\Theta(\delta,\mu,\mu,\rho\lambda) \geq \Theta(\delta,\mu,\mu,\lambda) \quad \text{and}$$

$$\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda) \geq \max\left\{\begin{array}{c} \Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \\ \Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \{\alpha\Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda)\} + \beta\max\left\{\begin{array}{c} \Upsilon(\mathcal{B}\eta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \\ \Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Upsilon(\mathcal{A}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Upsilon(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \Upsilon(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \Upsilon(\mathcal{T}\mu,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \\ \Upsilon(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda), \Upsilon(\mathcal{T}\mu,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \{\alpha\Upsilon(\mathcal{S}\delta,\mathcal{B}\mu,\mathcal{B}\mu,\lambda)\} + \beta\max\left\{\begin{array}{c} \Upsilon(\mathcal{B}\mu,\mathcal{T}\mu,\mathcal{T}\mu,\lambda), \\ \Upsilon(\mathcal{S}\delta,\mathcal{A}\delta,\mathcal{A}\delta,\lambda), \\ \Upsilon(\mathcal{S}\delta,\mathcal{T}\mu,\mathcal{T}\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Upsilon(\delta,\mu,\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Upsilon(\delta,\mu,\mu,\lambda), \Upsilon(\delta,\delta,\delta,\lambda), \Upsilon(\mu,\mu,\mu,\lambda), \\ \Upsilon(\delta,\mu,\mu,\lambda), \Upsilon(\mu,\delta,\delta,\lambda), \\ \{\alpha\Upsilon(\delta,\mu,\mu,\lambda)\} + \beta\max\left\{\begin{array}{c} \Upsilon(\mu,\mu,\mu,\lambda), \\ \Upsilon(\delta,\delta,\delta,\lambda), \\ \Upsilon(\delta,\mu,\mu,\lambda) \end{array}\right\} \end{array}\right\}$$

$$\Upsilon(\delta,\mu,\mu,\rho\lambda) \geq \max\left\{\begin{array}{c} \Upsilon(\delta,\mu,\mu,\lambda), 0, 0, \Upsilon(\delta,\mu,\mu,\lambda), \Upsilon(\mu,\delta,\delta,\lambda), \\ \{\alpha\Upsilon(\delta,\mu,\mu,\lambda)\} + \beta\max\{0,0,\Upsilon(\delta,\mu,\mu,\lambda)\} \end{array}\right\}$$

$$\Upsilon(\delta,\mu,\mu,\rho\lambda) \geq \Upsilon(\delta,\mu,\mu,\lambda)$$

Then by lemma (2.9), we get $\delta = \mu$. □

Corollary 3.1. Let $(\sum, \Xi, \Theta, \Upsilon, *, \Diamond)$ be the complete NMS and let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ be self mapping of $\sum$. Let the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$ be OWC and $\rho > 1$, then

$$\int_0^{\Xi(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)} \phi(\lambda)d\lambda \leq \int_0^{\min\left\{\begin{array}{c} \left(\begin{array}{c} \Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda), \Xi(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda), \\ \Xi(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \Xi(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda), \\ \Xi(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda) \end{array}\right) \\ \frac{\alpha\Xi(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Xi(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Xi(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma} \\ \frac{1+\Xi(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2} \end{array}\right\}} \phi(\lambda)d\lambda$$

$$\int_0^{\Theta(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)} \Psi(\lambda)d\lambda \geq \int_0^{\max\left\{\begin{array}{c}\left(\begin{array}{c}\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Theta(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Theta(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Theta(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Theta(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Theta(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Theta(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}\\ \frac{1+\Theta(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}} \Psi(\lambda)d\lambda$$

$$\int_0^{\Upsilon(\mathcal{A}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\rho\lambda)} \varphi(\lambda)d\lambda \geq \int_0^{\max\left\{\begin{array}{c}\left(\begin{array}{c}\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda),\\ \Upsilon(\mathcal{T}\eta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\Upsilon(\mathcal{S}\zeta,\mathcal{B}\eta,\mathcal{B}\eta,\lambda),\\ \Upsilon(\mathcal{T}\eta,\mathcal{A}\zeta,\mathcal{A}\zeta,\lambda)\end{array}\right)\\ \frac{\alpha\Upsilon(\mathcal{A}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)+\beta\Upsilon(\mathcal{B}\eta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)+\Gamma\Upsilon(\mathcal{S}\zeta,\mathcal{T}\eta,\mathcal{T}\eta,\lambda)}{\alpha+\beta+\Gamma}\\ \frac{1+\Upsilon(\mathcal{A}\zeta,\mathcal{S}\zeta,\mathcal{S}\zeta,\lambda)}{2}\end{array}\right\}} \varphi(\lambda)d\lambda$$

for all $\zeta,\eta \in \sum$ and $\lambda > 0$ such that $\mathcal{A}\omega = \mathcal{S}\omega = \omega$ and a unique point $\delta \in \sum$ such that $\mathcal{B}\delta = \mathcal{T}\delta = \delta$. Moreover $\delta = \omega$, so that there is a unique common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$.

4. Conclusion

In this paper, We explored new results in the notion of Neutrosophic Metric Space due to Kirisci, Simsek. We extend and generalize Al. Thagfi et al. paper in neutrosophic version and proved fixed point results in Occasionally weakly compatible mappings.

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