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On quasi hemi-slant submanifolds of LP-cosymplectic manifolds

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Abstract. In this paper, we define and study quasi hemi-slant submanifolds of Lorentzian almost contact metric manifolds. We mainly concern with quasi hemi-slant submanifolds of LP-cosymplectic manifolds. First we find conditions for integrability of distributions involved in the definition of quasi hemi-slant submanifolds of LP-cosymplectic manifolds. Further, we investigate the necessary and sufficient conditions for quasi hemi-slant submanifolds of LP-cosymplectic manifolds to be totally geodesic and geometry of foliations are determined.

1. Introduction

As a generalization of both holomorphic and totally real submanifolds, Chen [6] introduced the notion of slant submanifold of an almost Hermitian manifold. Later, such submanifolds have been studied by several geometers in the context of different ambient manifolds (see [2, 3, 5, 7, 8, 9, 13]). Further, as a generalization of CR-submanifolds and slant submanifolds Papaghiuc [11] introduced a new class of submanifolds called semi-slant submanifolds and Carriazo [4] introduced the notion of bi-slant submanifolds. The study of these types of submanifolds has shown interest by several geometers due to its application in quantum mechanics, mathematical physics as well as in computer design and image processing.

Recently, R. Prasad et.al. [12] defined and studied a new class of submanifold called quasi hemi-slant submanifold as a generalization of all the above mentioned submanifolds by considering ambient manifolds as a cosymplectic manifold. In this study authors have found out the necessary and sufficient conditions for integrability of invariant, anti-invariant and slant distributions. Further, authors have shown

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totally geodesicness of quasi hemi-slant submanifold of cosymplectic manifold under some conditions.

Motivated by the study of above authors in this article we define and study quasi hemi-slant submanifold of an LP-cosymplectic manifold. We investigate the conditions for integrability of invariant, anti-invariant and slant distributions which are involved in the definition of quasi hemi-slant submanifold of LP-cosymplectic manifold. Also we have studied geometry of fibres.

The paper is organized as follows: We recall basic formulas and definitions of LP-cosymplectic manifolds and their submanifolds in followed section. In section 3, we give definition of quasi hemi-slant submanifold of LP-cosymplectic manifold. Section 4 is devoted to investigating necessary and sufficient conditions for the geometry of distributions. In section 5, we check the geometry of fibers. Finally, we provide a non-trivial example of quasi hemi-slant submanifold of LP-cosymplectic manifold.

2. Preliminaries

A Lorentzian para contact metric manifold is a differentiable manifold $\tilde{M}$ of dimension $(2n+1)$, equipped with a structure (ϕ, ξ, η, g), where ϕ is a $(1,1)$-tensor field, η ia a 1-form, ξ is a vector field and g is a Lorentzian metric with signature $(-,+,+,...,+)$ satisfying [10]:

$$\phi^2 = I - \eta \otimes \xi,\ \eta(\xi) = -1,\ \phi\xi = 0,\ \eta \cdot \phi = 0, \tag{1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi), \tag{2}$$

for all vector fields X, Y in $\tilde{M}$. Moreover ϕ is symmetric with respect to g. A Lorentzian para-contact metric structure is called a Lorentzian para-cosymplectic structure if $\tilde{\nabla}\phi = 0$, where $\tilde{\nabla}$ denotes the Levi-Civita connection with respect to g. The manifold $\tilde{M}$ in this case is called a Lorentzian para-cosymplectic [in short LP-cosymplectic] manifold [10]. From the relation $\tilde{\nabla}\phi = 0$, it follows that $\tilde{\nabla}\xi = 0$.

Let M be a submanifold of a Lorentzian para contact metric manifold $\tilde{M}$ and the induced Lorentzian metric on M is denoted by the same symbol g throughout this paper, then the Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \tag{3}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{4}$$

for any $X, Y \in TM$ and $N \in T^{\perp}M$, where ∇ is the induced connection on M, $\nabla^{\perp}$ is the connection in the normal bundle, σ is the second fundamental form and A_N is the Weingarten endomorphism associated with N. Moreover A_N and σ are related by

$$g(\sigma(X,Y), N) = g(A_N X, Y).$$

For any $X \in TM$, we can write

$$\phi X = TX + FX, \tag{5}$$

where TX and FX are the tangential and normal components of ϕX on M, respectively. Similarly, for any $N \in T^{\perp}M$, we have

$$\phi N = tN + fN, \tag{6}$$

where tN and fN are the tangential and normal components of ϕN on M, respectively.

A submanifold M of a LP-cosymplectic manifold $\tilde{M}$ is said to be totally umbilical if

$$\sigma(X,Y) = g(X,Y)H.$$

If $\sigma(X,Y) = 0$ for all $X, Y \in TM$, then M is said to be totally geodesic and if $H = 0$, then M is called a minimal submanifold.

The covariant derivatives of the tensor fields T, F, t and f are defined as

$$\begin{aligned}
(\tilde{\nabla}_X T)Y &= \nabla_X TY - T\nabla_X Y, \\
(\tilde{\nabla}_X F)Y &= \nabla^{\perp}_X FY - F\nabla_X Y, \\
(\tilde{\nabla}_X t)N &= \nabla_X tN - t\nabla^{\perp}_X N, \\
(\tilde{\nabla}_X f)N &= \nabla^{\perp}_X fN - f\nabla^{\perp}_X N,
\end{aligned}$$

for any $X, Y \in TM$ and $N \in T^{\perp}M$.
The geometry of different classes of submanifolds of LP-cosymplectic manifolds have been studied by many geometers (see [**14, 15, 16**]) and references therein.

3. Quasi hemi-slant submanifolds of LP- cosymplectic manifolds

In this section, we define and study quasi hemi-slant submanifolds of LP-cosymplectic manifolds.

Definition 3.1. A submanifold M of a Lorentzian contact metric manifold is said to be quasi hemi-slant submanifold if there exist distributions $D, D^{\perp}$ and D^{θ} such that

(1) TM admits the orthogonal direct composition as

$$TM = D \oplus D^{\perp} \oplus D^{\theta} \oplus <\xi> .$$

(2) The distribution D is ϕ invariant i.e., $\phi D = D$
(3) The distribution $D^{\perp}$ is ϕ anti-invariant, i.e., $\phi D^{\perp} \subset T^{\perp}M$.
(4) For any non-zero vector field $X \in (D^{\theta})_x$, $x \in M$, the angle θ between ϕX and $(D^{\theta})_x$ is constant and independent of the choice of point x and X in $(D^{\theta})_x$.

In this case θ is called as a quasi hemi-slant angle of M. Suppose the dimension of distributions D, $D^{\perp}$ and D^{θ} are m_1, m_2 and m_3, respectively. Then we can easily see the following particular cases:

(1) If $m_1 = 0$, then M is a hemi-slant submanifold;

(2) If $m_2 = 0$, then M is a semi-slant submanifold;
(3) If $m_3 = 0$, then M is a semi-invariant submanifold.

We say that a quasi hemi-slant submanifolds is proper if $D \neq 0, D^\perp \neq 0$ and $\theta \neq 0, \frac{\pi}{2}$.
The notion of quasi hemi-slant submanifold is a generalization of invariant, anti-invariant, semi-invariant, slant, semi-slant and hemi-slant submanifolds.
Let M be a quasi hemi-slant submanifold of a Lorentzian contact metric manifold $\tilde{M}$. Then for any $X \in \Gamma(TM)$, we have

$$X = PX + QX + RX + \eta(X)\xi, \tag{7}$$

where P, Q and R denotes the projections on the distributions $D, D^\perp$ and D^θ respectively.
Now we put

$$\phi X = TX + FX, \tag{8}$$

where TX and FX are tangential and normal components of ϕX on M. Now making use of (7) and (8), we obtain

$$\phi X = TPX + FQX + TRX + FRX, \tag{9}$$

here since $\phi D = D$ and $\phi D^\perp \subset T^\perp M$, we have $FPX = 0$ and $TQX = 0$.
Hence, for any $X \in \Gamma(TM)$, it is easy to see that

$$TX = TPX + TRX$$

and

$$FX = FQX + FRX.$$

Thus from (9), we have

$$\phi(TM) = D \oplus FD^\perp \oplus TD^\theta \oplus FD^\theta$$

and

$$T^\perp M = FD^\perp \oplus FD^\theta \oplus \mu,$$

where μ is the orthogonal complement of $FD^\perp \oplus FD^\theta$ in $T^\perp M$ and it is invariant with respect to ϕ. For any $N \in T^\perp M$, we have

$$\phi N = tN + fN, \tag{10}$$

where $tN \in \Gamma(D^\perp \oplus D^\theta)$ and $fN \in \Gamma(\mu)$.
By using (1), (8) and (10), we obtain the following Lemma:

Lemma 3.1. *Let M be a quasi hemi-slant submanifold of a Lorentzian contact metric manifold $\tilde{M}$. Then, we have*

(1) $T^2 + tF = I + \eta \otimes \xi$ *and* $FT + fF = 0$ *on* TM;
(2) $Ft + f^2 = I$ *and* $Tt + tf = 0$ *on* $T^\perp M$.

With the help of (8) and (10), we obtain the following Lemma:

Lemma 3.2. *Let M be a quasi hemi-slant submanifold of a Lorentzian contact metric manifold $\tilde{M}$. Then, we have*

(1) $T^2X = \cos^2\theta X$;
(2) $g(TX, TY) = \cos^2\theta g(X, Y)$;
(3) $g(FX, FY) = \sin^2\theta g(X, Y)$.

for any $X, Y \in D^\theta$.

Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. Then for any X, $Y \in TM$, we have

$$\begin{aligned} \nabla_X TY - A_{FY}X - T\nabla_X Y - t\sigma(X, Y) &= 0, \\ \sigma(X, TY) + \nabla^\perp_X FY - F\nabla_X Y - f\sigma(X, Y) &= 0 \end{aligned}$$

and

$$TD = D, TD^\perp = 0,\ TD^\theta = D^\theta,\ tFD^\theta = D^\theta,\ tFD^\perp = D^\perp.$$

Lemma 3.3. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$, then we have*

$$A_{\phi Z}W = A_{\phi W}Z - T([W, Z]) \text{ and } \nabla^\perp_Z \phi W - \nabla^\perp_W \phi Z = F([Z, W]),$$

for all $Z, W \in D^\perp$.

Proof. Let $Z, W \in D^\perp$, then

$$(\tilde{\nabla}_Z \phi)W = 0,$$

which implies that

$$\tilde{\nabla}_Z \phi W - \phi(\tilde{\nabla}_Z W) = 0.$$

By using (3) and (4), we get

$$-A_{\phi W}Z + \nabla^\perp_Z \phi W - \phi[\nabla_Z W + \sigma(Z, W)] = 0.$$

Taking into account of (8) and (10) in the above equation, we obtain

$$-A_{\phi W}Z + \nabla^\perp_Z \phi W - T(\nabla_Z W) - F(\nabla_Z W) - t\sigma(Z, W) - f\sigma(Z, W) = 0.$$

Comparing tangential and normal components, we get

$$-A_{\phi W}Z - T(\nabla_Z W) - t\sigma(Z, W) = 0, \tag{11}$$
$$\nabla^\perp_Z \phi W - F(\nabla_Z W) - f\sigma(Z, W) = 0. \tag{12}$$

From equations (11) and (12), one can easily get the statement of Lemma (3.3). □

Lemma 3.4. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$, then*

(1) $g([X, Y], \xi) = 0$;
(2) $g(\tilde{\nabla}_X Y, \xi) = 0$.

for all $X, Y \in (D \oplus D^\perp \oplus D^\theta)$.

4. Integrability of distributions

In this section, we find some necessary and sufficient conditions for integrability of the distributions involved in the definition of quasi hemi-slant submanifolds of LP-cosymplectic manifolds.
First, we have the following theorem;

Theorem 4.1. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. The invariant distribution D is integrable if and only if*

$$g(\nabla_X TY - \nabla_Y TX, TRZ) = g(\sigma(Y, TX) - \sigma(X, TY), FQZ + FRZ),$$

for any $X, Y \in \Gamma(D)$ and $Z \in \Gamma(D^\perp \oplus D^\theta)$.

Proof. The distribution D is integrable on M if and only if

$$g([X, Y], \xi) = 0 \text{ and } g([X, Y], Z) = 0,$$

for any $X, Y \in \Gamma(D)$ and $Z \in \Gamma(D^\perp \oplus D^\theta)$. Since M is an LP-cosymplectic manifold, from Lemma (3.4) we have $g([X, Y], \xi) = 0$. Thus D is integrable if and only if $g([X, Y], Z) = 0$.
Now for any $X, Y \in \Gamma(D)$ and $Z = QZ + RZ \in \Gamma(D^\perp \oplus D^\theta)$, by using (2) we obtain

$$g([X, Y], Z) = g(\tilde{\nabla}_X \phi Y, \phi Z) - g(\tilde{\nabla}_Y \phi X, \phi Z).$$

Taking into account of (3) and (9) in the above equation, we get

$$g([X, Y], Z) = g(\nabla_X TY - \nabla_Y TX, TRZ) + g(\sigma(X, TY) - \sigma(Y, TX), FQZ + FRZ),$$

which completes the proof. □

Theorem 4.2. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. The anti-invariant distribution $D^\perp$ is integrable if and only if*

$$g(T([Z, W]), TX) = g(\nabla^\perp_Z FW - \nabla^\perp_W FZ, FQX),$$

for any $Z, W \in \Gamma(D^\perp)$ and $X \in \Gamma(D \oplus D^\theta)$.

Proof. The distribution $D^\perp$ on M is integrable if and only if

$$g([Z, W], \xi) = 0 \text{ and } g([Z, W], X) = 0,$$

for any $Z, W \in \Gamma(D^\perp)$, $X = PX + RX \in \Gamma(D \oplus D^\theta)$ and $\xi \in TM$. The first case is trivial. Thus $D^\perp$ is integrable if and only if $g([Z, W], X) = 0$.
Now for any $Z, W \in \Gamma(D^\perp)$, $X = PX + RX \in \Gamma(D \oplus D^\theta)$, by using (2) we obtain

$$g([Z, W], X) = g(\tilde{\nabla}_Z \phi W, \phi X) - g(\tilde{\nabla}_W \phi Z, \phi X).$$

Using (4), (9) and taking into account of Lemma (3.3)(i) in the above equation, we get

$$g([Z, W], X) = g(\nabla^\perp_Z FW - \nabla^\perp_W FZ, FQX) - g(T[Z, W], TX),$$

which gives the assertion. □

Theorem 4.3. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. The slant distribution D^θ is integrable if and only if*

$$g(A_{FW}Z - A_{FZ}W, TPX) = g(A_{FTW}Z - A_{FTZ}W, X) + g(\nabla^\perp_Z FW - \nabla^\perp_W FZ, FQX),$$

for any $Z, W \in \Gamma(D^\theta)$ and $X \in \Gamma(D \oplus D^\perp)$.

Proof. The distribution D^θ on M is integrable if and only if

$$g([Z,W],\xi) = 0 \textit{ and } g([Z,\, W], X) = 0,$$

for any $Z, W \in \Gamma(D^\theta)$, $X \in \Gamma(D \oplus D^\perp)$ and $\xi \in TM$. The first case is trivial. Thus its enough to prove that $g([Z,W],X) = 0$.

Now for any $Z, W \in \Gamma(D^\theta)$, $X = PX + QX \in \Gamma(D \oplus D^\perp)$, by using (2) we obtain

$$g([Z,W],X) = g(\tilde{\nabla}_Z \phi W, \phi X) - g(\tilde{\nabla}_W \phi Z, \phi X).$$

Using (9) in the above equation we have

$$g([Z,W],X) = g(\tilde{\nabla}_Z FW, \phi X) + g(\tilde{\nabla}_Z \phi TW, X) - g(\tilde{\nabla}_W FZ, \phi X) - g(\tilde{\nabla}_W \phi TZ, X).$$

Taking into account of (3), (4), (9) and Lemma (3.3)(i), we obtain

$$\begin{aligned} g([Z,W],X) &= g(A_{FZ}W - A_{FW}Z, \phi X) + g(\nabla^\perp_Z FW - \nabla^\perp_W FZ, \phi X) \\ &\quad + \cos^2\theta g([Z,W],X) - g(A_{FTZ}W - A_{FTW}Z, X). \end{aligned}$$

Which implies

$$\begin{aligned} \sin^2\theta g([Z,W],X) &= g(A_{FZ}W - A_{FW}Z, TPX) + g(\nabla^\perp_Z FW - \nabla^\perp_W FZ, FQX) \\ &\quad - g(A_{FTZ}W - A_{FTW}Z, X). \end{aligned}$$

This completes the proof. □

From Theorem (4.3), we have the following sufficient conditions for the slant distribution D^θ to be integrable:

Corollary 4.4. *Let M be a quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. If*

$$\begin{aligned} \nabla^\perp_Z FW - \nabla^\perp_W FZ &\in FD^\theta \oplus \mu, \\ A_{FTW}Z - A_{FTZ}W &\in D^\theta \textit{ and}, \\ A_{FZ}W - A_{FW}Z &\in D^\perp \oplus D^\theta, \end{aligned}$$

for any $Z, W \in \Gamma(D^\theta)$, then the slant distribution D^θ is integrable.

5. Totally geodesic foliations

In this section, we investigate the geometry of foliations of a quasi hemi-slant submanifold of LP-cosymplectic manifold. Also, we find conditions for the totally geodesicness.

Theorem 5.1. *Let M be a proper quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. Then M is totally geodesic if and only if*

$$g(\sigma(X,PY)+\cos^2\theta\sigma(X,RY),U)=g(A_{FQY}X+A_{FRY}X,tU)-g(\nabla^{\perp}_X FTRY,U)-g(\nabla^{\perp}_X FY,fU),$$

for any $X,Y\in\Gamma(TM)$ and $U\in\Gamma(T^{\perp}M)$.

Proof. For any $X,Y\in\Gamma(TM)$ and $U\in\Gamma(T^{\perp}M)$, by virtue of (7) we have

$$g(\tilde{\nabla}_X Y,U)=g(\tilde{\nabla}_X PY,U)+g(\tilde{\nabla}_X QY,U)+g(\tilde{\nabla}_X RY,U).$$

Using (2), we get

$$g(\tilde{\nabla}_X Y,U)=g(\tilde{\nabla}_X\phi PY,\phi U)+g(\tilde{\nabla}_X\phi QY,\phi U)+g(\tilde{\nabla}_X TRY,\phi U)+g(\tilde{\nabla}_X FRY,\phi U).$$

Applying (8) and by virtue of (2), we obtain

$$\begin{aligned} g(\tilde{\nabla}_X Y,U) &= g(\tilde{\nabla}_X PY,U)+g(\tilde{\nabla}_X\phi FQY,\phi U)+g(\tilde{\nabla}_X T^2RY,U)\\ &\quad +g(\tilde{\nabla}_X FTRY,U)+g(\tilde{\nabla}_X FRY,\phi U). \end{aligned}$$

Using (3), (4) and Lemma (8), we have

$$\begin{aligned} g(\tilde{\nabla}_X Y,U) &= g(\sigma(X,PY),U)+g(\nabla^{\perp}_X FY,fU)-g(A_{FQY}X+A_{FRY}X,tU)\\ &\quad +\cos^2\theta g(\sigma(X,RY),U)+g(\nabla^{\perp}_X FTRY,U). \end{aligned}$$

Hence the proof. □

Theorem 5.2. *Let M be a proper quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. Then anti-invariant distribution $D^{\perp}$ defines totally geodesic foliation if and only if*

$$g(\nabla^{\perp}_X\phi Y,FRZ)=g(A_{\phi Y}X,TPZ+TRZ)\ \text{and}\ g(A_{\phi Y}X,tV)=g(\nabla^{\perp}_X\phi Y,fV),$$

for any $X,Y\in\Gamma(D^{\perp})$, $Z\in\Gamma(D\oplus D^{\theta})$ and $V\in(T^{\perp}M)$.

Proof. For any $X,Y\in\Gamma(D^{\perp})$, $Z=PZ+RZ\in\Gamma(D\oplus D^{\theta})$. Using (2) we have

$$g(\tilde{\nabla}_X Y,Z)=g(\tilde{\nabla}_X\phi Y,\phi Z).$$

By virtue of (9) and by using (4), we obtain

$$g(\tilde{\nabla}_X Y,Z)=-g(A_{\phi Y}X,TPZ+TRZ)+g(\nabla^{\perp}_X\phi Y,FRZ). \tag{13}$$

Again, let $X,Y\in\Gamma(D^{\perp}))$ and $V\in\Gamma(T^{\perp}M)$, then we have

$$g(\tilde{\nabla}_X Y,Z)=g(\tilde{\nabla}_X\phi Y,\phi V).$$

Using (2) and (3), we have

$$g(\tilde{\nabla}_X Y, V) = -g(A_{\phi Y} X, tV) + g(\nabla^{\perp}_X \phi Y, fV). \quad (14)$$

From equations (13) and (14), it is clear that $D^{\perp}$ defines totally geodesic foliation if and only if $g(A_{\phi Y} X, TPZ + TRZ) = g(\nabla^{\perp}_X \phi Y, FRZ)$ and $g(A_{\phi Y} X, tV) = g(\nabla^{\perp}_X \phi Y, fV)$. This completes the proof of the theorem. □

Theorem 5.3. *Let M be a proper quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. Then the slant distribution D^{θ} defines a totally geodesic foliation on M if and only if*

$$\begin{aligned} g(\nabla^{\perp}_X FY, FQZ) &= g(A_{FY} X, TPZ) + g(A_{FTY} X, Z) \text{ and} \\ g(A_{FY} X, tV) &= g(\nabla^{\perp}_X FY, fV) + g(\nabla^{\perp}_X FTY, V), \end{aligned}$$

for any $X, Y \in \Gamma(D^{\theta})$, $Z \in \Gamma(D \oplus D^{\perp})$ and $V \in (T^{\perp} M)$.

Proof. For any $X, Y \in \Gamma(D^{\theta})$, $Z \in \Gamma(D \oplus D^{\perp})$, using (2) we have

$$g(\tilde{\nabla}_X Y, Z) = g(\tilde{\nabla}_X \phi Y, \phi Z).$$

Using (3), (4), (7), (8) and Lemma (3.2) we have

$$\sin^2 \theta g(\tilde{\nabla}_X Y, Z) = g(\nabla^{\perp}_X FY, FQZ) - g(A_{FTY} X, X) - g(A_{FY} X, TPZ). \quad (15)$$

Similarly, we obtain

$$\sin^2 \theta g(\tilde{\nabla}_X Y, V) = g(\nabla^{\perp}_X FY, fV) + g(\nabla^{\perp}_X FTY, V) - g(A_{FY} X, tV). \quad (16)$$

Thus the theorem proof follows from the equations (15) and (16). □

Theorem 5.4. *Let M be a proper quasi hemi-slant submanifold of an LP-cosymplectic manifold $\tilde{M}$. Then the invariant distribution D defines a totally geodesic foliation on M if and only if*

$$\begin{aligned} g(\nabla_X TY, TRZ) &= -g(\sigma(X, TY), NQZ + NRZ), \text{and} \\ g(\nabla_X TY, tU) &= -g(\sigma(X, TY), fU), \end{aligned}$$

for any $X, Y \in \Gamma(D^{\perp})$, $Z \in \Gamma(D \oplus D^{\theta})$ and $V \in (T^{\perp} M)$.

6. Example

Let $\tilde{M}$ be a 15-dimensional manifold

$$\tilde{M} = \{(x_i, y_i, z) = x_1, x_2, \cdots, x_7, y_1, y_2, \cdots, y_7) \in R^{15}\}.$$

If we define

$$e_i = \frac{\partial}{\partial x_i}, \; e_{7+i} = \frac{\partial}{\partial y_i}, \; e_{15} = \xi = -\frac{\partial}{\partial z} \, for \, i = 1, 2, \cdots 7.$$

Let g be a Lorentzian metric defined by

$$(dx_i)^2 + (dy_j)^2 - \eta \otimes \eta.$$

Then we find that $g(e_i, e_i) = -1$ and $g(e_i, e_j) = 0$ for $1 \leq i \neq j \leq 15$.
Hence $\{e_1, e_2, \cdots e_{15}\}$ forms an orthonormal basis. Thus 1-form $\eta = dz$ is defined by $\eta(e_i) = g(e_i, \xi)$ for any $e_i \in \Gamma(T\tilde{M})$.
We define (1,1)-tensor field ϕ as

$$\phi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial x_i},\ \phi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial y_j},\ \phi(\frac{\partial}{\partial z}) = 0, \forall i, j = 1, 2, \cdots, 7.$$

By using linearity of ϕ and g, we have

$$\begin{aligned} \phi^2 &= I - \eta \otimes \xi,\ \phi\xi = 0,\ \eta(\xi) = -1, \\ g(\phi X, \phi Y) &= g(X, Y) + \eta(X)\eta(Y), \end{aligned}$$

for any $X, Y \in \Gamma(T\tilde{M})$. Hence $(\tilde{M}, \phi, \xi, \eta, g)$ is a Lorentzian almost paracontact metric manifold. Further, we can easily show that $(\tilde{M}, \phi, \xi, \eta, g)$ is an LP-cosymplectic manifold of dimension 15.

Now, assume that M is an immersed submanifold of $\tilde{M}$ given by

$$\Omega(u, v, w, r, s, t, q) = (u, w, 0, \frac{s}{\sqrt{2}}, 0, \frac{t}{\sqrt{2}}, 0, v, r\cos\theta, rsin\theta, 0, \frac{s}{\sqrt{2}}, 0, \frac{t}{\sqrt{2}}, q),$$

where $0 < \theta < \frac{\pi}{2}$. Then one can easily see that the tangent bundle of M is spanned by the vectors

$$\begin{aligned} W_1 &= \frac{\partial}{\partial x_1},\ W_2 = \frac{\partial}{\partial y_1},\ W_3 = \frac{\partial}{\partial x_2},\ W_4 = \cos\theta\frac{\partial}{\partial y_2} + \sin\theta\frac{\partial}{\partial y_3}, \\ W_5 &= \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_4} + \frac{\partial}{\partial y_5}\right),\ W_6 = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_6} + \frac{\partial}{\partial y_7}\right),\ W_7 = \frac{\partial}{\partial z}. \end{aligned}$$

Then using Lorentzian para contact structure of $\tilde{M}$, we have

$$\begin{aligned} \phi W_1 &= \frac{\partial}{\partial x_1},\ \phi W_2 = -\frac{\partial}{\partial y_1},\ \phi W_3 = \frac{\partial}{\partial x_2},\ \phi W_4 = -\left(\cos\theta\frac{\partial}{\partial y_2} + \sin\theta\frac{\partial}{\partial y_3}\right), \\ \phi W_5 &= \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_4} - \frac{\partial}{\partial y_5}\right),\ \phi W_6 = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_6} - \frac{\partial}{\partial y_7}\right),\ \phi W_7 = 0. \end{aligned}$$

Now, let the distributions $D = span\{W_1, W_2\}$, $D^\theta = span\{W_3, W_4\}$, $D^\perp = span\{W_5, W_6\}$. It is easy to see that D is invariant, D^θ is slant with slant angle θ and $D^\perp$ is anti-invariant.

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